A Combined Methodology of H_∞ Fuzzy Tracking Control and Virtual Reference Model for a PMSM

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Abstract. *The aim of this paper is to present a new fuzzy tracking strategy for a permanent magnet synchronous machine (PMSM) by using Takagi-Sugeno models (T-S). A feedback-based fuzzy control with H_∞ tracking performance and a concept of virtual reference model are combined to develop a fuzzy tracking controller capable to track a reference signal and ensure a minimum effect of disturbance on the PMSM system. First, a T-S fuzzy model is used to represent the PMSM nonlinear system with disturbance. Next, an integral fuzzy tracking control based on the concept of virtual desired variables (VDVs) is formulated to simplify the design of the virtual reference model and the control law. Finally, based on this concept, a two-stage design procedure is developed: i) determine the VDVs from the nonlinear system output equation and generalized kinematics constraints ii) calculate the feedback controller gains by solving a set of linear matrix inequalities (LMIs). Simulation results are provided to demonstrate the validity and the effectiveness of the proposed method.*

Keywords

H_∞ tracking performance, integral action, LMIs, PMSM, Takagi-Sugeno fuzzy model, virtual desired variables.

1. Introduction

The permanent magnet synchronous machine (PMSM) drives are widely used in industrial applications such as production tools, computer numerically controlled machines, chip mounted devices, robots and hard disk drives. They are receiving increased attention because of their high efficiency, high power/weight and torque/inertia ratios. However, their analysis and control is a difficult task, due to the inherent nonlinearities and load torque. Thus, the linear control method cannot ensure satisfactory performances. In order to overcome the associated difficulties in the design of a controller for PMSM, several schemes have been proposed in the last three decades, e.g. adaptive control [1], neural network control [2], nonlinear feedback linearization control [3], sliding mode control [4], [5]. Recently, many design methods based on the fuzzy control theory have been proposed to deal with the problem of tracking control for PMSM [6], [7]. We propose here a new fuzzy tracking control for PMSM based on T-S fuzzy models [8] by taking into account the variations of load torque.

During the last years, the problem of tracking control of nonlinear systems using T–S fuzzy models has been studied by many authors [9], [10], [11], [12], [13], [14]. It has become popular because of its efficiency in controlling nonlinear systems. Its main property is to describe the local dynamics by linear local models where the output of the global model is obtained by fuzzy blending of these linear models through nonlinear fuzzy membership functions. The fuzzy tracking control of nonlinear systems aims to ensure the best tracking between the output of the nonlinear system and the reference. In [9], the tracking problem of nonlinear systems has been solved using a synthesis of both the fuzzy control and the linear multivariable control theories. However, in [10], a novel concept of virtual desired variables has been proposed to simplify the design of the reference model and the control law. Based on this concept, the tracking control problem can be converted into a stabilization problem which has been treated by several researches using Lyapunov approach [15], [16]. The stability analysis of a T-S fuzzy system needs a symmetric positive definite matrix to satisfy a set of LMIs which can be solved efficiently by convex programming techniques.

The fuzzy control of the PMSM based on the concept of VDVs has been treated in [17], [18], [19], but without taking into account the variations of load torque, which represents the disturbance effect in the system. Given that, in real industrial applications, the synchronous machine is always affected by different disturbances; their presence deteriorates the tracking control performance. Hence, many works have been done to design robust control strategies for T-S fuzzy models. For example, in [11], a robust fuzzy tracking controller based on internal model principle has been introduced to track a reference signal. On the other hand, in [12] and [13], the reference input is considered as a disturbance and is attenuated using a robust criterion. However, in [14] a H_∞ tracking control has been introduced to deal with the robust performance design problem of nonlinear systems.

The objective of this work is to develop a new state feedback controller for a PMSM based on the concept of VDVs and the H_∞ tracking control. In this case, the proposed controller is able to drive the state of the synchronous machine to track a specific desired reference and to reject a completely unknown disturbance. First, the PMSM system with disturbance is represented by a T-S fuzzy model. Next, an integral fuzzy tracking control based on a set of VDVs is formulated to simplify the design of the model reference and control law. Finally, the tracking control performance of the augmented fuzzy system is analysed by the Lyapunov's method based on H_∞ control which can be formulated into LMI problems. Simulations are carried out on PMSM in order to verify the effectiveness of the proposed methodology.

2. Problem Formulation

2.1. Mathematical Model of PMSM

The dynamic model of the synchronous machine in $d-q$ reference frame can be described by the following nonlinear system [20], [21]:

$$\dot{\vec{x}}(t) = f(\vec{x}(t)) + g(\vec{x}(t))\vec{u}(t) + \vartheta(\vec{x}(t))w(t), \quad (1)$$

where

$$f(\vec{x}(t)) = \begin{bmatrix} -\frac{B_f}{J}\omega(t) + \frac{3p\lambda}{2J}i_q(t) \\ -\frac{p\lambda}{L_q}\omega(t) - \frac{R}{L_q}i_q(t) - p\omega(t)i_d(t) \\ p\omega(t)i_q(t) - \frac{R}{L_d}i_d(t) \end{bmatrix},$$

$$g(\vec{x}(t)) = \begin{bmatrix} 0 & 0 \\ \frac{1}{L_q} & 0 \\ 0 & \frac{1}{L_d} \end{bmatrix}, \quad \vartheta(\vec{x}(t)) = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix},$$

$$\vec{x}(t) = \begin{bmatrix} \omega & i_q & i_d \end{bmatrix}^T, \quad \vec{u}(t) = \begin{bmatrix} u_q & u_d \end{bmatrix}^T,$$

in which ω is the rotor speed, w is the load torque (the load torque is an exogenous disturbance), (i_q, i_d) are the current components in the d-q axis, (u_q, u_d) are the stator voltage components in the $d-q$ axis, (L_d, L_q) are the stator inductors in the $d-q$ axis. The machine parameters are: the stator winding resistance R, the moment of inertia of the rotor J, the friction coefficient relating to the rotor speed B_f, the flux linkage of the permanent magnets λ, the number of poles pairs p.

In our work, the smooth-air-gap of the synchronous machine systems are considered, i.e., $L_q = L_d = L$.

2.2. T-S Fuzzy Model of PMSM

In order to express the nonlinear model of the machine as a T-S model with the measurable parameter (speed) as decision variable, we rewrite Eq. (1) in the following nonlinear state space form:

$$\begin{cases} \dot{\vec{x}}(t) = \mathbf{A}(\omega(t))\vec{x}(t) + \mathbf{B}\vec{u}(t) + \mathbf{D}w(t) \\ \vec{y}(t) = \varphi(\vec{x}(t)) = \mathbf{C}\vec{x}(t), \end{cases} \quad (2)$$

where:

$$\mathbf{A}(\omega(t)) = \begin{bmatrix} -\frac{B_f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\omega(t) \\ 0 & p\omega(t) & -\frac{R}{L} \end{bmatrix}, \quad w(t) = \mathbf{C}_r(t),$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix}.$$

Assuming that the speed is bounded as: $\underline{\omega} \leq \omega(t) \leq \overline{\omega}$ and using the well-known sector nonlinearity approach [22], the nonlinear system of the machine Eq. (1) can be described by a T-S model with $r = 2^1$ fuzzy If-Then rules as follows:

Rule 1 : **If** $z(t)$ is F_{11}, **Then**

$$\dot{\vec{x}}(t) = \mathbf{A}_1\vec{x}(t) + \mathbf{B}_1\vec{u}(t) + \mathbf{D}_1 w(t).$$

Rule 2 : **If** $z(t)$ is F_{12}, **Then**

$$\dot{\vec{x}}(t) = \mathbf{A}_2\vec{x}(t) + \mathbf{B}_2\vec{u}(t) + \mathbf{D}_2 w(t),$$

where $z(t) = \omega(t)$ is the premise variable, F_{11} and F_{12} are the membership functions which can be defined as:

$$F_{11}(\omega(t)) = \frac{\omega(t) - \underline{\omega}}{\overline{\omega} - \underline{\omega}},\ F_{12}(\omega(t)) = 1 - F_{11}. \quad (3)$$

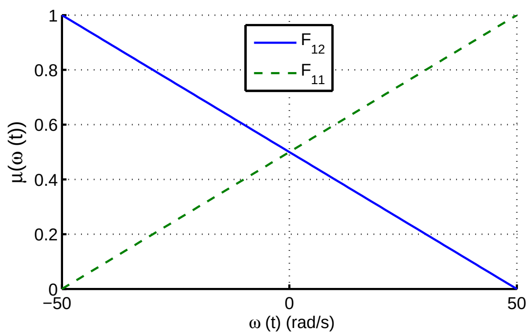


Fig. 1: Membership functions of the decision variable.

The matrices of the local models can be defined as:

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{B_f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\overline{\omega} \\ 0 & p\overline{\omega} & -\frac{R}{L} \end{bmatrix}, \mathbf{D}_1 = \mathbf{D}_2 = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix}.$$

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{B_f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\underline{\omega} \\ 0 & p\underline{\omega} & -\frac{R}{L} \end{bmatrix}, \mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}.$$

Using the product-inference rule, singleton fuzzifier, and the centre of gravity defuzzifier, the above fuzzy rules base is inferred as follows:

$$\dot{\vec{x}}(t) = \sum_{i=1}^{r} h_i(z(t))(\mathbf{A}_i\vec{x}(t) + \mathbf{B}_i\vec{u}(t) + \mathbf{D}_i w(t)), \quad (4)$$

where

$$h_i(z(t)) = \frac{F_{1i}(z(t))}{\sum_{j=1}^{r} F_{1j}((\omega(t))}, \quad (5)$$

for all $t > 0$, $h_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$.

3. Fuzzy Tracking Controller Design

Our goal is to design a fuzzy controller capable of driving the state of the system $\vec{x}(t)$ to track a specified set of VDVs $\vec{x}_d(t)$ and minimizing the effect of disturbance on the machine. The feedback tracking control is required to satisfy:

$$\vec{x}(t) - \vec{x}_d(t) \to 0 \quad as \quad t \to \infty. \quad (6)$$

According to $\vec{y}(t) = \varphi(\vec{x}(t))$, it is natural to require $\vec{y}_d(t) = \varphi(\vec{x}_d(t))$, which denotes the desired output state. Now, let $\tilde{\vec{x}}(t) = \vec{x}(t) - \vec{x}_d(t)$ be defined as the tracking error and its time derivative is given by:

$$\dot{\tilde{\vec{x}}}(t) = \dot{\vec{x}}(t) - \dot{\vec{x}}_d(t). \quad (7)$$

Replacing Eq. (4) by its value in Eq. (7) and adding the term $\sum_{i=1}^{r} h_i(z(t))\mathbf{A}_i(\vec{x}_d(t) - \vec{x}_d(t))$, Eq. (7) becomes:

$$\begin{aligned}\dot{\tilde{\vec{x}}}(t) = \sum_{i=1}^{r} h_i(z)(\mathbf{A}_i\tilde{\vec{x}}(t) + \mathbf{D}_i w(t) + \mathbf{B}_i\vec{u}(t) \\ + \mathbf{A}_i\vec{x}_d(t)) - \dot{\vec{x}}_d(t)\end{aligned} \quad (8)$$

In Eq. (8), if we introduce new variable $\vec{\tau}(t)$ that satisfy the following relation:

$$\begin{aligned}\sum_{i=1}^{r} h_i(z(t))\mathbf{B}_i\vec{\tau}(t) = \sum_{i=1}^{r} h_i(z(t))\mathbf{B}_i\vec{u}(t) + \\ \sum_{i=1}^{r} h_i(z(t))\mathbf{A}_i\vec{x}_d(t) - \dot{\vec{x}}_d(t),\end{aligned} \quad (9)$$

where $\vec{\tau}(t)$ is a new controller which will be designed based on Parallel Distributed Compensation (PDC) technique [15]. Using Eq. (9), the tracking error system Eq. (8) can be rewritten as follows:

$$\dot{\tilde{\vec{x}}}(t) = \sum_{i=1}^{r} h_i(z(t))(\mathbf{A}_i\tilde{\vec{x}}(t) + \mathbf{B}_i\vec{\tau}(t) + \mathbf{D}_i w(t)). \quad (10)$$

The new local state feedback controllers are designed to deal with the tracking control problem as:

Rule 1: **If** $z(t)$ is F_{11} **Then** $\vec{\tau}(t) = -\mathbf{K}_1\tilde{x}(t)$,
Rule 2: **If** $z(t)$ is F_{12} **Then** $\vec{\tau}(t) = -\mathbf{K}_2\tilde{x}(t)$,

where $z(t) = \omega(t)$. The final output of the fuzzy controller is determined by the summation:

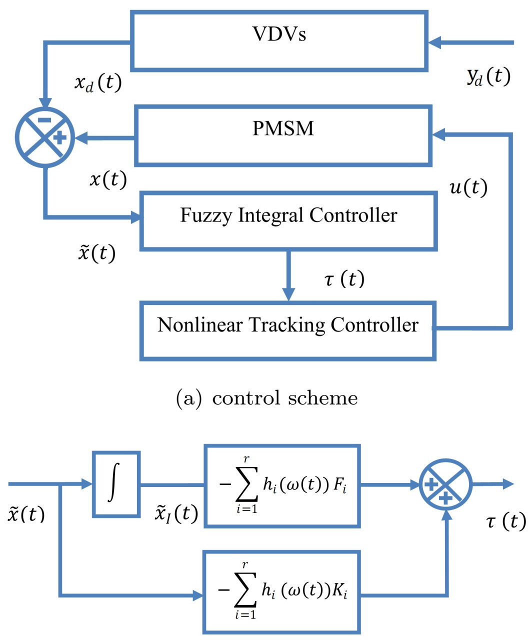
$$\vec{\tau}(t) = -\sum_{i=1}^{r} h_i(z(t))\mathbf{K}_i\tilde{\vec{x}}(t), \qquad i = 1, ..., r = 2. \quad (11)$$

In order to reject slow varying disturbances according to the PMSM, an integral action is added as shown in Fig. 2(b). The new fuzzy controller $\vec{\tau}(t)$ can be rewritten as:

$$\vec{\tau}(t) = -\sum_{i=1}^{r} h_i(z(t))\mathbf{K}_i\tilde{\vec{x}} - \sum_{i=1}^{r} h_i(z(t))\mathbf{F}_i\tilde{\vec{x}}_I, \quad (12)$$

where

$$\tilde{\vec{x}}_I(t) = \int_0^{t_f} \tilde{\vec{x}}(t)\mathrm{d}t.$$



(a) control scheme

(b) fuzzy integral controller

Fig. 2: Fuzzy integral tracking control scheme.

From Eq. (12), it can be rewritten:

$$\vec{\tau}(t) = -\sum_{i=1}^{r} h_i(z(t)) \begin{bmatrix} \mathbf{K}_i & \mathbf{F}_i \end{bmatrix} \begin{bmatrix} \vec{\tilde{x}}(t) \\ \vec{\tilde{x}}_I(t) \end{bmatrix}.$$

Thus, the new controller $\vec{\tau}(t)$ is:

$$\vec{\tau}(t) = -\sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{K}}_i \vec{\bar{X}}(t). \qquad (13)$$

The augmented T-S fuzzy model with an integral action can be written in the following form:

$$\dot{\vec{\bar{X}}} = \sum_{i=1}^{r} h_i(z(t))(\bar{\mathbf{A}}_i \vec{\bar{X}}(t) + \bar{\mathbf{B}}_i \vec{\tau}(t) + \bar{\mathbf{D}}_i w(t)). \qquad (14)$$

$$\bar{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i & 0 \\ \mathbf{I} & 0 \end{bmatrix}, \ \bar{\mathbf{B}}_i = \begin{bmatrix} \mathbf{B}_i \\ 0 \end{bmatrix}, \ \bar{\mathbf{D}}_i = \begin{bmatrix} \mathbf{D}_i \\ 0 \end{bmatrix}.$$

In the case $\mathbf{B}_1 = \mathbf{B}_2 =, ..., \mathbf{B}_r = \mathbf{B}$, the augmented T-S fuzzy model can be written as:

$$\dot{\vec{\bar{X}}}(t) = \sum_{i=1}^{r} h_i(z(t))[\mathbf{G}_i \vec{\bar{X}}(t) + \bar{\mathbf{D}}_i w(t)], \qquad (15)$$

where

$$\mathbf{G}_i = \bar{\mathbf{A}}_i - \bar{\mathbf{B}}\bar{\mathbf{K}}_i, \ \ \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}.$$

4. H_∞ Tracking Control Design

The purpose of present work is to design a fuzzy state feedback controller in Eq. (13), for the augmented system Eq. (15), capable to drive the state of the PMSM system to track the desired variables $\vec{x}_d(t)$ and guaranteed a minimum effect of disturbance on the PMSM. The presence of $w(t)$ will deteriorate the control performance of the control system. In order to minimize the effects of $w(t)$ on the control system, the following H_∞ performances related to tracking error have been considered [23], [24]:

$$\int_0^\infty \vec{\bar{X}}^T(t)\vec{\bar{X}}(t)\mathrm{d}t \le \gamma^2 \int_0^\infty w^T(t)w(t)\mathrm{d}t, \qquad (16)$$

where γ is a prescribed value, which denotes the worst case effect of disturbance $w(t)$ on $\vec{\bar{X}}(t)$. The results of H_∞ norm bounded are given in following Lem. 1:

Lemma 1. *The augmented fuzzy system described by Eq. (15), if there exists* $\mathbf{X}^T = \mathbf{X} > 0$ *common solution of the following matrix inequalities:*

$$\begin{bmatrix} \bar{\mathbf{A}}_i\mathbf{X} + \mathbf{X}\bar{\mathbf{A}}_i^T - \bar{\mathbf{B}}\mathbf{M}_i - \mathbf{M}_i^T\bar{\mathbf{B}}^T & \bar{\mathbf{D}}_i & \mathbf{X} \\ \bar{\mathbf{D}}_i^T & -\gamma^2\mathbf{I} & 0 \\ \mathbf{X} & 0 & -\mathbf{I} \end{bmatrix} < 0. \qquad (17)$$

for all $i = 1, ..., r$.

Then, the H_∞ tracking control performance in Eq. (16) is guaranteed for a prescribed γ via the fuzzy controller Eq. (13). The control gains are given by:

$$\bar{\mathbf{K}}_i = \mathbf{M}_i\mathbf{X}^{-1}. \qquad (18)$$

Proof. Consider the Lyapunov function $V(\vec{\bar{X}}(t)) = \vec{\bar{X}}^T(t)\mathbf{P}\vec{\bar{X}}(t)$ where $\mathbf{P} = \mathbf{P}^T > 0$ the common positive matrix. The time derivative of $V(\vec{\bar{X}}(t))$ will be required to satisfy the following condition:

$$V(\vec{\bar{X}}(t)) < 0. \qquad (19)$$

In order to achieve the H_∞ tracking performance related to the tracking error, for $\vec{x}_d$, Eq. (19) becomes:

$$\dot{V}(\vec{\bar{X}}(t)) + \vec{\bar{X}}^T(t)\vec{\bar{X}}(t) - \gamma^2 w^T(t)w(t) < 0. \qquad (20)$$

Replacing $V(\vec{\bar{X}}(t))$ by its value $\vec{\bar{X}}^T\mathbf{P}\vec{\bar{X}}$ in Eq. (20), the last equation can be written as the following form LMI:

$$\sum_{i=1}^{r} h_i(z(t)) \left\{ \vec{\bar{X}}^T(\mathbf{G}_i\mathbf{P} + \mathbf{P}\mathbf{G}_i^T)\vec{\bar{X}} + w^T \begin{bmatrix} \bar{\mathbf{D}}_i^T & \mathbf{P} \end{bmatrix} \vec{\bar{X}} \right\}$$
$$+ \sum_{i=1}^{r} h_i(z(t))\vec{\bar{X}}^T \begin{bmatrix} \mathbf{P} & \bar{\mathbf{D}}_i \end{bmatrix} w - \gamma^2 w^T w < 0. \qquad (21)$$

This main:

$$\sum_{i=1}^{r} h_i(z(t)) \begin{bmatrix} \vec{\bar{X}}^T & w^T \end{bmatrix} \begin{bmatrix} \mathbf{G}_i\mathbf{P} + \mathbf{P}\mathbf{G}_i^T & \bar{\mathbf{D}}_i \\ \bar{\mathbf{D}}_i^T & -\gamma^2\mathbf{I} \end{bmatrix}$$
$$\begin{bmatrix} \vec{\bar{X}} \\ w \end{bmatrix} < 0. \qquad (22)$$

From Eq. (22), we can write:

$$\begin{bmatrix} \sum_{i=1}^{r} h_i(z(t))(\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T)+\mathbf{I} & \mathbf{P}\sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i \\ \sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} \end{bmatrix} < 0. \tag{23}$$

$$\begin{bmatrix} \sum_{i=1}^{r} h_i(z(t))(\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T) & \mathbf{P}\sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i \\ \sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix} < 0. \tag{24}$$

From Eq. (24), we can write:

$$\begin{bmatrix} \sum_{i=1}^{r} h_i(z(t))(\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T) & \mathbf{P}\sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i \\ \sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}\begin{bmatrix} \mathbf{I} & 0 \end{bmatrix} < 0. \tag{25}$$

Using the Schur's complement, Eq. (25) is equivalent to:

$$\begin{bmatrix} \sum_{i=1}^{r} h_i(z(t))(\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T) & \mathbf{P}\sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i & \mathbf{I} \\ \sum_{i=1}^{r} h_i(z(t))\bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} & 0 \\ \mathbf{I} & 0 & -\mathbf{I} \end{bmatrix} < 0. \tag{26}$$

From Eq. (26), we can write:

$$\begin{bmatrix} (\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T) & \mathbf{P}\bar{\mathbf{D}}_i & \mathbf{I} \\ \bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} & 0 \\ \mathbf{I} & 0 & -\mathbf{I} \end{bmatrix} < 0. \tag{27}$$

After congruence with $diag\left(\begin{bmatrix} \mathbf{P}^{-1} & \mathbf{I} & \mathbf{I} \end{bmatrix}\right)$, inequality Eq. (27) becomes:

$$\begin{bmatrix} \mathbf{P}^{-1} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} (\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T) & \mathbf{P}\bar{\mathbf{D}}_i & \mathbf{I} \\ \bar{\mathbf{D}}_i^T\mathbf{P} & -\gamma^2\mathbf{I} & 0 \\ \mathbf{I} & 0 & -\mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{P}^{-1} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} < 0. \tag{28}$$

Developed the last equation, Eq. (28) can be written as:

$$\begin{bmatrix} \mathbf{P}^{-1}(\mathbf{G}_i\mathbf{P}+\mathbf{P}\mathbf{G}_i^T)\mathbf{P}^{-1} & \mathbf{P}^{-1}\mathbf{P}\bar{\mathbf{D}}_i & \mathbf{P}^{-1} \\ \bar{\mathbf{D}}_i^T\mathbf{P}\mathbf{P}^{-1} & -\gamma^2\mathbf{I} & 0 \\ \mathbf{P}^{-1} & 0 & -\mathbf{I} \end{bmatrix} < 0. \tag{29}$$

Considering $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{M}_i = \mathbf{K}_i\mathbf{X}$, we obtain the same matrix as in Eq. (17):

$$\begin{bmatrix} \bar{\mathbf{A}}_i\mathbf{X}+\mathbf{X}\bar{\mathbf{A}}_i^T-\bar{\mathbf{B}}\mathbf{M}_i-\mathbf{M}_i^T\bar{\mathbf{B}}^T & \bar{\mathbf{D}}_i & \mathbf{X} \\ \bar{\mathbf{D}}_i^T & -\gamma^2\mathbf{I} & 0 \\ \mathbf{X} & 0 & -\mathbf{I} \end{bmatrix} < 0. \tag{30}$$

5. VDVs and Control Law Design

In order to determine the VDVs $\vec{x}_d(t)$ and control law $\vec{u}(t)$, we use Eq. (9) which is rewritten below:

$$\sum_{i=1}^{r} h_i(z(t))\mathbf{B}_i(\vec{u}(t)-\vec{\tau}(t)) = -\sum_{i=1}^{r} h_i(z(t)) \mathbf{A}_i\vec{x}_d(t)+\dot{\vec{x}}_d(t). \tag{31}$$

Assuming that:

$$g(x) = \sum_{i=1}^{r} h_i(z(t))\mathbf{B}_i, \quad A(x) = \sum_{i=1}^{r} h_i(z(t))\mathbf{A}_i.$$

Then, the equation Eq. (31) can be rewritten as the following compact form:

$$g(x)(\vec{u}(t)-\vec{\tau}(t)) = -A(x)\vec{x}_d(t)+\dot{\vec{x}}_d(t)). \tag{32}$$

By applying Eq. (32) to the PMSM model, we obtain the following matrix form:

$$\begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}(\vec{u}(t)-\vec{\tau}(t)) = -\begin{bmatrix} -\frac{B_f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\omega \\ 0 & p\omega & -\frac{R}{L} \end{bmatrix} \cdot \begin{bmatrix} \omega_d \\ i_{qd} \\ i_{dd} \end{bmatrix} + \begin{bmatrix} \dot{\omega}_d \\ \dot{i}_{qd} \\ \dot{i}_{dd} \end{bmatrix}, \tag{33}$$

where $\vec{\tau} = [\tau_q \quad \tau_d]^T$ is the new controller to be designed via LMIs approach; $\vec{x}_d = [\omega_d \quad i_{qd} \quad i_{dd}]^T$ is the vector of the desired state.

According to the first equation of Eq. (33), it follows that:

$$\dot{\omega}_d = -\frac{B_f}{J}\omega_d + \frac{3p\lambda}{2J}i_{qd}, \tag{34}$$

which induces that:

$$i_{qd} = (\dot{\omega}_d + \frac{B_f}{J}\omega_d)\frac{2J}{3p\lambda}. \tag{35}$$

Note that for a PMSM there is no need for a flow model. As a result, the position of the rotor is the

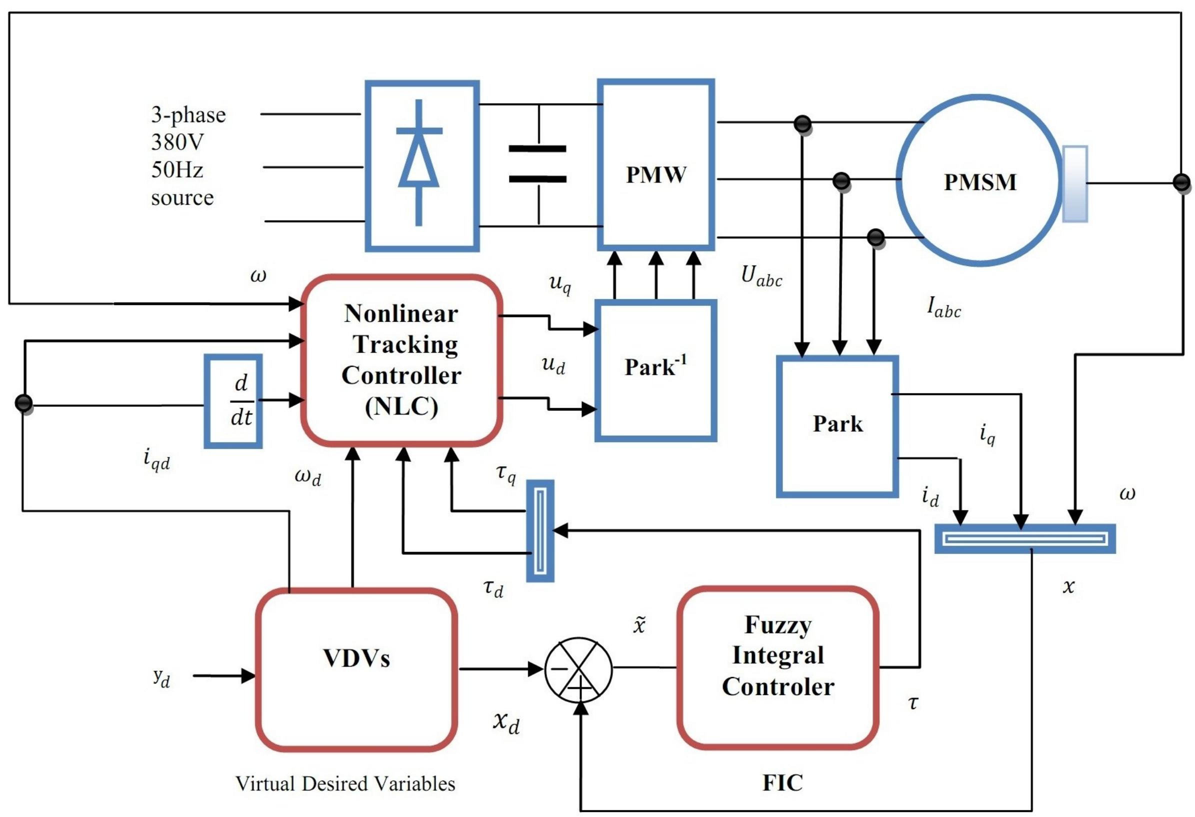


Fig. 3: Diagram of the fuzzy tracking control of the PMSM.

angle of reference. Furthermore, as we have a smooth poles machine, the best choice for its operation is obtained for a value where the internal angle is equal $\frac{\pi}{2}$ that means $i_{dd} = 0$. Consequently, we obtained the following vector of the desired state:

$$\vec{x}_d(y_d) = \begin{bmatrix} \omega_d \\ i_{qd} \\ i_{dd} \end{bmatrix} = \begin{bmatrix} y_d \\ (\dot{y}_d + \dfrac{B_f}{J} y_d)\dfrac{2J}{3p\lambda} \\ 0 \end{bmatrix}, \quad (36)$$

where y_d is the desired speed. From the second and the third equations of Eq. (33), we obtain the control input:

$$\begin{cases} u_q = p\lambda\omega_d + Ri_{qd} + L\dot{i}_{qd} + Lp\omega i_{dd} + \tau_q \\ u_d = -pL\omega i_{qd} + Ri_{dd} + L\dot{i}_{dd} + \tau_d. \end{cases} \quad (37)$$

Replacing i_{dd} by its value in Eq. (37), we obtain the fallowing control voltage:

$$\begin{cases} u_q = p\lambda\omega_d + Ri_{qd} + L\dot{i}_{qd} + \tau_q \\ u_d = -pL\omega i_{qd} + \tau_d. \end{cases} \quad (38)$$

6. Simulation Results

In this section, simulation tests have been carried out on PMSM to verify the effectiveness of the proposed method using the schematic diagram of the fuzzy tracking control given in Fig. 3, which has three main blocks: The VDVs block, the Fuzzy Integral Controller (FIC) block and the Nonlinear Tracking Controller (NTC) block. The first block computes the vector of VDVs based on the desired speed, which will be used by the blocks: FIC and NTC, the second block calculates the new control law based on the fuzzy control and the objective of the last block is to generate the control voltages that it will attack the PMSM via the inverse Park and Pulse Width Modulated (PWM) elements. Note that the NTC block needs some states that come from VDVs block and the machine.

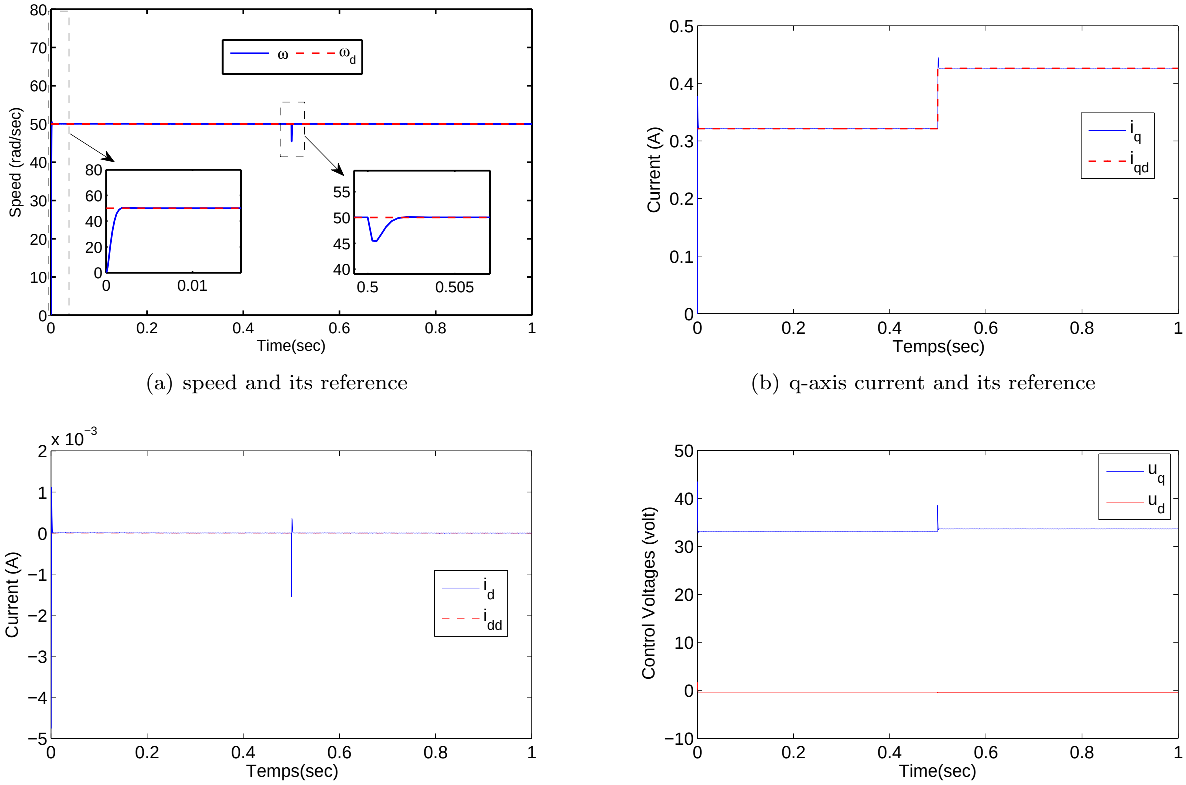
Using the Lem. 1 and the parameters of the PMSM listed in Tab. 1, the following control gains are obtained:

$$\mathbf{K}_1 = \begin{bmatrix} 3.8664 & 8.7633 & 0.0718 \\ -0.2105 & -0.4954 & 0.2480 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} 3.8582 & 8.7454 & 0.0876 \\ 0.2775 & 0.6448 & 0.2588 \end{bmatrix},$$

$$\mathbf{F}_1 = \begin{bmatrix} 2.9331 & 0.0192 & -0.2939 \\ 0.1920 & -0.0093 & 1.1998 \end{bmatrix},$$

$$\mathbf{F}_2 = \begin{bmatrix} 2.9395 & 0.0143 & 0.2797 \\ -0.1441 & -0.0112 & 1.2043 \end{bmatrix}.$$



(a) speed and its reference

(b) q-axis current and its reference

(c) d-axis current and its reference

(d) control voltage

Fig. 4: Simulation results for speed set-point ($y_d = 50$ rad·s^{-1}) with a load torque applied at $t = 0.5$ s.

Tab. 1: Parameter values of the PMSM.

Parameter	Value	Unity
Rated power	300	W
Moment of inertia J	$6.36 \cdot 10^{-4}$	kg·m^2
Stator resistance R	4.55	Ω
Stator inductance L	11.6	mH
Flux linkage λ	0.317	V·s·rad^{-1}
Friction coefficient B_f	$6.11 \cdot 10^{-3}$	N·m·s·rad^{-1}
Number of poles pair p	2	–
Speed bound ω	$-50 \leq \omega \leq 50$	rad·s^{-1}

The proposed scheme is verified in two cases:

6.1. Speed Regulation

Consider speed regulation of the desired speed $y_d = 50$ rad·s^{-1} with a load torque $w = 5$ N·m applied at $t = 0.5$ s, the initial state is set to be $x(0) = 0$. The simulation results for the desired and actual speeds, desired and actual q-axis currents, desired and actual d-axis currents and control voltage are shown in Fig. 4(a), Fig. 4(b), Fig. 4(c) and Fig. 4(d), respectively, which demonstrate that the time response of the regulation control is very low, also the tracking error is very small until the appearance of the disturbance at $t = 0.5$ s where only a little discrepancy become clear for a laps of time before its rejection by the controller.

Furthermore, the less speed, current and voltage tracking errors, the better the tracking performance, highlights the good performances of the proposed fuzzy control method in terms of tracking and disturbances rejection. Results demonstrate that the machine system with the synthesized fuzzy controller has a good behavior. Indeed, the speed and d-q axis currents track well the reference trajectory with good reliability over the whole speed range. From Fig. 4(b), Fig. 4(c) and Fig. 4(d), it is clear that the current and voltage responses are in the expected ranges.

Figure 5 indicates clearly the good tracking performance in the case of low desired speed.

6.2. Sinusoidal Speed Tracking

Consider the sinusoidal speed tracking for $y_d(t) = 50\sin(t)$ rad·s^{-1} and a load torque $w = 5$ N·m applied at $t = 1$ s, the initial state is set to be $x(0) = [40\ \ 0\ \ 0]^T$. The simulation results for the desired and actual speeds, desired and actual q-axis currents, desired and actual d-axis currents and input control voltage are shown in Fig. 6(a), Fig. 6(b), Fig. 6(c) and Fig. 6(d), respectively, Fig. 6 shows that time response

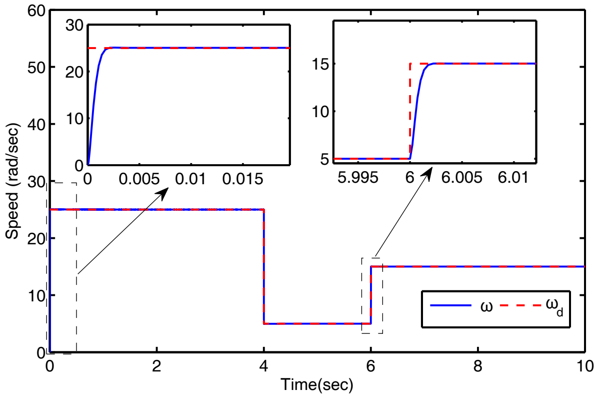


Fig. 5: Response of PMSM for variable speed setpoint.

of the tracking is very low, the tracking error is apparent, but very small and the system is robust to the disturbance with only a little deviation during the disturbance is applied.

Moreover, the proposed controller is compared with the fuzzy state feedback controller developed in [10] which has been applied to the PMSM in many works like [17], [18], [19]. The objective is to force the output speed of the PMSM to track the step reference $y_d = 40$ rad·s^{-1}, in both controllers. Thus, the feedback control gains obtained from the Theorem 1 [10] are:

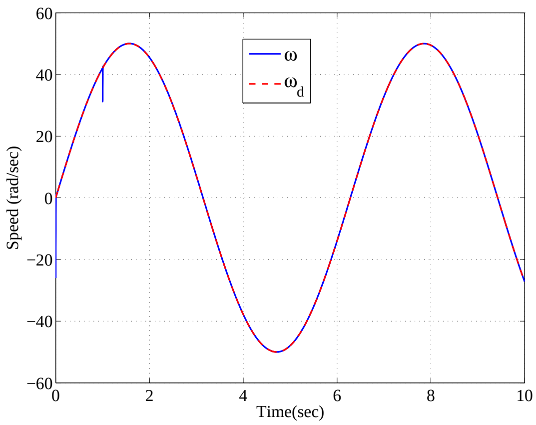
$$\mathbf{K}_1 = \begin{bmatrix} 6.4802 & 7.4405 & -0.3584 \\ -0.4546 & -0.5098 & 0.0852 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} 6.4941 & 7.4719 & -0.1526 \\ 0.0083 & 0.0114 & 0.0526 \end{bmatrix}.$$

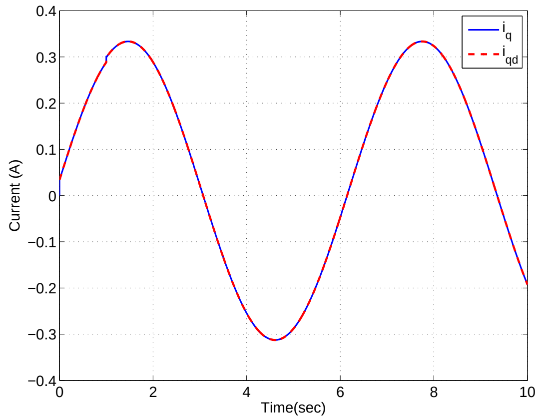
with the following diagonal positive matrix:

$$\mathbf{D} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

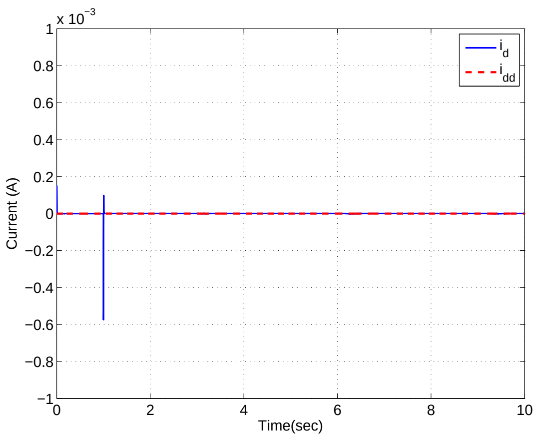
We applied the proposed state feedback controller and the compared controller to the PMSM system (1) using the same machine parameters listed in Tab. 1, the initial value is set to be: $x(0) = 0$. The simulation result is depicted in Fig. 7: speed setpoint (dashed



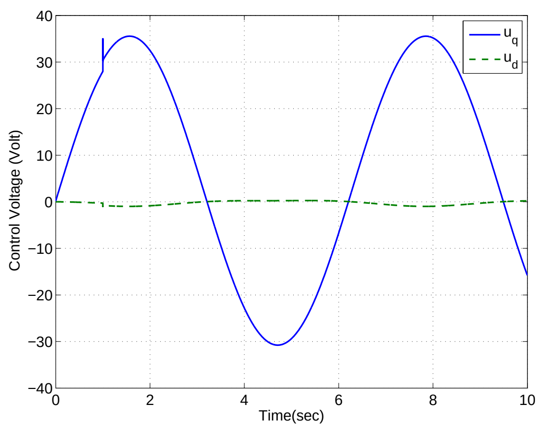
(a) speed and its reference



(b) q-axis current and its reference



(c) d-axis current and its reference



(d) control voltage

Fig. 6: Simulation results for sinusoidal desired speed ($y_d(t) = 50\sin(t)$ rad·s^{-1}) with a load torque applied at $t = 1$ s.

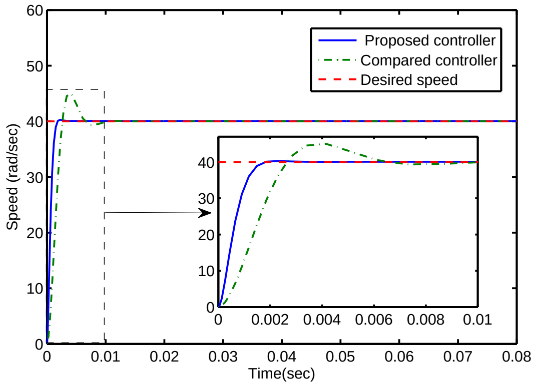


Fig. 7: Simulation results for the comparison.

red), speed responses for the proposed fuzzy control (solid blue) and for the compared controller (dash-dotted green), respectively.

It is clear that the actual speed of the tracking control system can follow its desired trajectory for both methods. Moreover, almost minimum time response is ensured with less overshoot for the proposed controller, as indicated in Fig. 7. To assess the performance of the proposed controller, we have also used the Root Mean Square Error (RMSE) between the output and its reference which can be defined by:

$$RMSE = \sqrt{\frac{\sum_{k=1}^{N} (y(k) - y_d(k))^2}{N}}. \quad (39)$$

The time response, the overshoot and the RMSE resulting from the proposed and the compared fuzzy tracking control are shown in Tab. 2, which demonstrate that the proposed control strategy has better tracking performance than that compared controller. In addition, the proposed controller is able to reject a completely the unknown disturbance.

Tab. 2: Comparison of the time response, overshoot and RMSE relative to the performance of the control strategies considered.

Controller	Proposed Co.	Compared Co.
Time response (s)	0.0014	0.0073
Overshoot (%)	0.59	11.13
RMSE	12.61	14.39

7. Conclusion

This paper outlines a new fuzzy integral tracking control scheme for nonlinear systems described by T-S fuzzy model. To this end, an integral control scheme and a fuzzy tracking controller based on VDVs has been combined to design a robust controller able to reject a completely unknown disturbance. Sufficient conditions for stability are derived from a Lyapunov's method based on H_∞ performances. The concept of VDVs has been used to simplify the design of the reference model and the control law. The controller has been tested successfully for a permanent magnet synchronous machine. The results show that the desired performances for the controlled system can be achieved via the proposed fuzzy tracking control method.

References

[1] LIU, T. H., H. T. PU and C. K. LIN. Implementation of an Adaptive Position Control System of a Permanent Magnet Synchronous Motor and its Application. *Power Electronics, IET*. 2010, vol. 4, iss. 2, pp. 121–130. ISSN 1751-8660. DOI: 10.1049/iet-epa.2009.0036.

[2] LIN, C. K., T. H. LIU and S. H. YANG. Nonlinear Position Controller Design with Input-Output Linearisation Technique for an Interior Permanent Magnet Synchronous Motor Control System. *Power Electronics, IET*. 2008, vol. 1, iss. 1, pp. 14–26. ISSN 1755-4535. DOI: 10.1049/iet-pel:20070177.

[3] EL-SOUSY, F. F. M. Hybrid H_∞-Based Wavelet Neural Network Tracking Control for Permanent Magnet Synchronous Motor Servo Drives, *IEEE Transactions on Industrial Electronics*. 2010, vol. 57, iss. 9, pp. 3157–3166. ISSN 0278-0046. DOI: 10.1109/TIE.2009.2038331.

[4] KAZRAJI, S. M. and M. B. B. SHARIFIAN. Direct Thrust Force and Flux Control of a PM-Linear Synchronous Motor Using Fuzzy Sliding-Mode Observer. *Advances in Electrical and Electronic Engineering*. 2015, vol. 13, iss. 1, pp. 1–9. ISSN 1804-3119. DOI: 10.15598/aeee.v13i1.999.

[5] BRIS, P., J. VITTEK and P. MAKYS. Position Control of PMSM in Sliding Mode. *Advances in Electrical and Electronic Engineering*. 2008, vol. 7, iss. 1, pp. 198–201. ISSN 1804-3119.

[6] CHOI, H. H., V. T. T. NGA and J. W. JUNG. Design and Implementation of a Takagi-Sugeno Fuzzy Speed Regulator for a Permanent Magnet Synchronous Motor. *IEEE Transactions on Industrial Electronics*. 2012, vol. 59, iss. 8, pp. 3069–3077. ISSN 0278-0046. DOI: 10.1109/TIE.2011.2141091.

[7] MAAMOUN, A., Y. M. ALSAVED and A. SHALTOUT. Fuzzy Logic Based Speed Controller for Permanent Magnet Synchronous Motor Drive. In: *IEEE International Conference on*

Mechatronics and Automation. Takamatsu: IEEE, 2013, pp. 1518–1522. ISBN 978-1-4673-5557-5. DOI: 10.1109/ICMA.2013.6618139.

[8] TAKAGI, T. and M. SUGENO. Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Transactions on Systems, Man, and Cybernetics, Part C*. 1985, vol. 15, iss. 1, pp. 116–132. ISSN 0018-9472. DOI: 10.1109/TSMC.1985.6313399.

[9] TSENG, C.-S., B. S. CHEN and H. J. UANG. Fuzzy Tracking Control Design for Nonlinear Dynamic Systems via T-S Fuzzy Model. *IEEE Transaction on Fuzzy Systems*. 2001, vol. 9, iss. 3, pp. 381–399. ISSN 1063-6706. DOI: 10.1109/91.928735.

[10] LIAN, K. Y. and J. LIOU. Output Tracking Control for Fuzzy Systems via Output Feedback Design. *IEEE Transactions on Fuzzy Systems*. 2006, vol. 14, iss. 5, pp. 381–392. ISSN 1083-4419. DOI: 10.1109/3477.826946.

[11] ZHANG, J., M. FEI, T. YANG and Y. TAN. Robust Fuzzy Tracking Control of Nonlinear Systems with Uncertainty via T-S Fuzzy Model. In: *Third international conference on Fuzzy Systems and Knowledge Discovery*. Xian: Springer Berlin Heidelberg, 2006, pp. 188–198. ISBN 978-3-540-45917-0. DOI: 10.1007/11881599_21.

[12] CHEN, B. S., C.-S. TSENG and H. J. UANG. Mixed H_2/H_∞ Fuzzy Output Feedback Control Design for Nonlinear Dynamic Systems: An LMI Approach. *IEEE Transaction on Fuzzy Systems*. 2000, vol. 8, iss. 3, pp. 249–265. ISSN 1063-6706. DOI: 10.1109/91.855915.

[13] TOULOTTE, P. F., S. DELPRAT, T. M. GUERRAA and J. BOONAERT. Vehicle Spacing Control using Robust Fuzzy Control with Pole Placement in LMI Region. *Engineering Applications of Artificial Intelligence*. 2008, vol. 21, iss. 5, pp. 756–768. ISSN 0952-1976. DOI: 10.1016/j.engappai.2007.07.009.

[14] KHIAR, D. Robust Takagi Sugeno Fuzzy Control of a Spark Ignition Engine. *Control Engineering Practice*. 2007, vol. 15, iss. 1, pp. 1446–1456. ISSN 0967-0661. DOI: 10.1016/j.conengprac.2007.02.003.

[15] TANAKA, K. and M. SANO. A Robust Stabilization Problem of Fuzzy Control Systems and its Application to Backing Up Control of a Truck-Trailer. *IEEE Transactions on Fuzzy Systems*. 1994, vol. 2, iss. 2, pp. 119–134. ISSN 1063-6706. DOI: 10.1109/91.277961.

[16] MANAI, Y. and Y. BENREJEB. New PDC Controller Design for Stabilization of Continuous Takagi-Sugeno Fuzzy System. In: 10^{th} *International Multi-Conference on Systems, Signals and Devices*. Hammamet: IEEE, 2013, pp. 1–4. ISBN 978-1-4673-6459-1. DOI: 10.1109/SSD.2013.6564116.

[17] LIAN, K. Y., C. H. CHIANG and H. W. TU. LMI-Based Sensorless Control of Permanent-Magnet Synchronous Motors. *IEEE Transactions on Industrial Electronics*. 2007, vol. 54, iss. 5, pp. 2769–2777. ISSN 0278-0046. DOI: 10.1109/TIE.2007.899829.

[18] JUNG, J. W., T. H. KIM and H. H. CHOI. Speed Control of a Permanent Magnet Synchronous Motor with a Torque Observer: a Fuzzy Approach. *IET Control Theory and Applications*. 2010, vol. 4, iss. 12, pp. 2971–2981. ISSN 1751-8644. DOI: 10.1049/iet-cta.2009.0469.

[19] OUNNAS, D., S. CHENIKHER and T. BOUKTIR. Tracking Control For Permanent Magnet Synchronous Machine Based On Takagi-Sugeno Fuzzy Models. In: 3^{rd} *International Conference on Ecological Vehicles and Renewable Energies*. Monte Carlo: IEEE, 2013, pp. 1–5. ISBN 978-1-4673-5269-7. DOI: 10.1109/EVER.2013.6521548.

[20] PARK, Z. L., Y. J. JOO, B. ZHANG and G. CHEN. Bifurcations and Chaos in a Permanent Magnet Synchronous Motor. *IEEE Transaction on Circuits and Systems: Fundamental Theory and Applications*. 2002, vol. 49, iss. 3, pp. 383–387. ISSN 1057-7122. DOI: 10.1109/81.989176.

[21] ZHAO, Y., T. ZHANQ and Q. SUN. Robust Fuzzy Control for Permanent Magnet Synchronous Motor Chaotic Systems with Uncertain Parameters. In: *International Conference on Automation and Logistics*. Shenyang: IEEE, 2009, pp. 7–5. ISBN 978-1-4244-4794-7. DOI: 10.1109/ICAL.2009.5262783.

[22] OHTAKE, H. O., K. TANAKA and H. WANG. Fuzzy Modeling via Sector Nonlinearity Concept. In: *IFSA World Congress and 20th NAFIPS International Conference*. Vancouver: IEEE, 2001, pp. 127–132. ISBN 0-7803-7078-3. DOI: 10.1109/NAFIPS.2001.944239.

[23] UANG, H. J. and G. S. HUANG. A Robust Fuzzy Model Following Observer-Based Control Design for Nonlinear System. In: *International Conference on Control Applications*. Hsin-Chu: IEEE, 2004, pp. 171–176. ISBN 0-7803-8633-7. DOI: 10.1109/CCA.2004.1387206.

[24] CHONG, L., Q. G. WANG and T. H. LEE. H_∞ Output Tracking Control for Nonlinear Via TS Fuzzy Model Approach. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*. 2006, vol. 36, iss. 2, pp. 450–457. ISSN 1083-4419. DOI: 10.1109/TSMCB.2005.856723.

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